

THREE-DIMENSIONAL PENETRATION SIMULATION USING SMOOTHED PARTICLE HYDRODYNAMICS

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This paper presents three-dimensional computational simulations of the hypervelocity impact (HVI) using standard smoothed particle hydrodynamics (SPH). The classic Taylor-Bar-Impact test is revisited with the focus on the variation of results corresponding to the different model parameters in the SPH implementation. The second example involves both normal and oblique HVIs of a sphere on the thin plate, producing large deformation of structures. Based on original experimental results and some numerical results reported previously, some comparisons are also made, in the hope of providing informative data on appropriate SPH implementation options for the software being developed. The results obtained show that the current SPH procedure is well suited for the HVI problems.

Keywords: Hypervelocity impact; numerical simulation; mesh-free method; SPH.

1. Introduction

For decades, mesh-based methods have dominated the numerical simulations for solving problems in engineering and science. However, they can, in some instances,

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suffer from difficulties in dealing with some problems such as fragmentation and crack propagation with moving discontinuities. It is apparent that their reliance on an underlying mesh is not well suited to those problems. Instead of viable strategies, there are certainly other ways, e.g. the so-called mesh-free methods, as detailed in the monograph by Liu [2002]. These mesh-free methods have attracted much attention in recent decades, mainly due to their flexibility in modeling the domain with only scattered nodes.

Among various mesh-free methods, SPH is the potential and promising alternative for many applications. Originally proposed by Monaghan [1992] for modeling astrophysical phenomena, SPH has been substantially improved for its applications to the areas of computational fluid dynamics and solid mechanics, e.g. [Swegle *et al.* (1995); Johnson and Beissel (1996); Libersky *et al.* (1993)]. A comprehensive introduction to the SPH method and its variations are given in the monograph by Liu and Liu [2003].

A recent program was initiated at the Centre for Advanced Computations for Engineering and Science (ACES), National University of Singapore, with the aim of developing a software package to perform SPH calculations for the explosion-related problems as well as hypervelocity impact (HVI) and penetration problems. An essential aspect of the developmental program involves the choice of the implementation options in various SPH applications. Some of the test results on simulating explosion of high explosives and underwater explosion are given in [Liu *et al.* (2003a, 2003b)]. This paper presents a three-dimensional simulation of hypervelocity impacts (both normal and oblique) of a sphere on the thin plate. The scope of the discussion hereby is limited to aspects pertaining only to SPH–HVI applications.

2. SPH Procedure

Starting with an integral representation of a function and its derivatives, the SPH method discretizes governing partial differential equations through the so-called kernel approximation and particle approximation. The kernel approximation allows the spatial gradient of a field function to be calculated from the values of appropriately defined smoothing function and its derivatives. The particle approximation is then performed to all terms in the equations using the concept of compact support with a finite number of particles, producing a set of discretized ordinary differential equations with respect to time. The above mesh-free and Lagrangian nature makes the SPH a good candidate of tools to solve HVI problem for materials with strength.

In HVI situations, the solid materials' behavior like fluids, which is governed by the conservation laws of hydrodynamics, together with Lagrangian description of the kinematic equation. Following a standard procedure [Libersky *et al.* (1993)], a set of SPH formulation can be written as follows:

$$\frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j (\mathbf{v}_i^\beta - \mathbf{v}_j^\beta) \frac{\partial W_{ij}}{\partial \mathbf{x}_i^\beta},\tag{1a}$$

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$$\frac{D\mathbf{v}_i^{\alpha}}{Dt} = -\sum_{j=1}^N m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right) \frac{\partial W_{ij}}{\partial \mathbf{x}_i^{\alpha}} + \sum_{j=1}^N m_j \left(\frac{\mathbf{S}_i^{\alpha\beta}}{\rho_i^2} + \frac{\mathbf{S}_j^{\alpha\beta}}{\rho_j^2}\right) \frac{\partial W_{ij}}{\partial \mathbf{x}_i^{\beta}},\tag{1b}$$

$$\frac{De_i}{Dt} = \frac{1}{2} \sum_{j=1}^{N} m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} \right) (\mathbf{v}_i^\beta - \mathbf{v}_j^\beta) \frac{\partial W_{ij}}{\partial \mathbf{x}_i^\beta} + \frac{1}{\rho_i} \mathbf{S}_i^{\alpha\beta} \dot{\varepsilon}_i^{\alpha\beta}, \tag{1c}$$

$$\frac{D\mathbf{x}_i}{Dt} = \mathbf{v}_i^{\alpha}.$$
 (1d)

The summation convention is adopted in Eq. (1). Dependent variables include the scalar density ρ , pressure p, specific internal energy e, velocity vector \mathbf{v}^{β} , strain rate tensor $\dot{\varepsilon}_{i}^{\alpha\beta}$ and traceless deviatoric stress tensor $\mathbf{S}_{i}^{\alpha\beta}$, with the spatial coordinates \mathbf{x} and time t being the independent variables. W herein is the particle kernel function and m is its mass. The equations are evolved as time steps forward in the moving Lagrangian frame.

To account for the artificial viscosity, such as Monaghan-type [Monaghan and Gingold (1983)], appropriately defined artificial viscosity terms are usually added to the physical pressure term. Thus, one must modify the velocity and energy evolution equations accordingly to incorporate this factor. Similarly, consideration of artificial heat, such as the one proposed by Monaghan [1995], will result in an inclusion of artificial heat term to the energy evolution equation. Taking into account both artificial terms, we can rewrite Eqs. (1b) and (1c) as given hereunder:

$$\frac{D\mathbf{v}_{i}^{\alpha}}{Dt} = -\sum_{j=1}^{N} m_{j} \left(\frac{p_{i}}{\rho_{i}^{2}} + \frac{p_{j}}{\rho_{j}^{2}} + \Pi_{ij} \right) \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\alpha}} + \sum_{j=1}^{N} m_{j} \left(\frac{\mathbf{S}_{i}^{\alpha\beta}}{\rho_{i}^{2}} + \frac{\mathbf{S}_{j}^{\alpha\beta}}{\rho_{j}^{2}} \right) \frac{\partial W_{ij}}{\partial \mathbf{x}_{i}^{\beta}}, \qquad (2a)$$

$$\frac{De_i}{Dt} = \frac{1}{2} \sum_{j=1}^{N} m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) (\mathbf{v}_i^\beta - \mathbf{v}_j^\beta) \frac{\partial W_{ij}}{\partial \mathbf{x}_i^\beta} + \frac{1}{\rho_i} \mathbf{S}_i^{\alpha\beta} \dot{\varepsilon}_i^{\alpha\beta} + H_i \,. \tag{2b}$$

It is worth noticing that in the above energy evolution equation (Eq. (2b)), the calculation of work done by the traceless deviatoric stress is valid only in the elastic range. However, the plastic yielding can be dominant in an HVI problem. Therefore, one must calculate incremental plastic work within every time step and then incorporate it into the energy evolution. For this calculation, a yield criterion, such as von Mises, is necessary. A simple way to calculate this plastic work is to first estimate the effective plastic strain increment $\Delta \varepsilon_{\text{eff}}^p$ as follows:

$$\Delta \varepsilon_{\text{eff}}^p = \frac{\sigma_{\text{eff}}^* - Y_0}{3G},\tag{3}$$

where σ_{eff}^* , Y_0 and G are the provisional von Mises flow stress, yield strength and shear modulus, respectively. While σ_{eff}^* is calculated from the deviatoric stress tensor $\mathbf{S}_i^{\alpha\beta}$, Y_0 is a given (but not necessarily constant) material property. One can then 674 C. E. Zhou, G. R. Liu & K. Y. Lou

estimate the incremental plastic work $\triangle W_p^n$ within the current time step n using the equation as follows:

$$\Delta W_p^n = \frac{1}{2} (\sigma_{\text{eff}}^{n+1} + \sigma_{\text{eff}}^n) \Delta \varepsilon_{\text{eff}}^p \left(\frac{m}{\rho^{n+1/2}}\right). \tag{4}$$

In Eq. (4), the term $\rho^{n+1/2}$ results from the use of central difference time integration scheme, indicating the density centered at $t^{n+1/2}$. In calculating the effective stress σ_{eff} , one may use the Radial Return method [Wilkins (1984)] to scale back the deviatoric stress to the yield surface.

In the course of solution, an appropriate material strength model, such as Johnson–Cook Model [Johnson and Cook (1983)], is of importance in the SPH application to solids. Also, an equation of state (EOS), such as Mie-Gruneisen EOS [Zukas (1990)], is required to establish the relationship between temperature, volume and pressure for a given substance.

It should be pointed out that the SPH approximation lacks interpolation completeness so that the linear consistency does not hold. In addition, the standard SPH algorithm using the cubic-spline kernel possesses an undesirable instability in the tensile regime when applied to solid mechanics. Some studies on remedies or improvements with regard to SPH's particle inconsistency and tensile instability can be found, in more detail, in Swegle *et al.* [1995]; Fulk [1997]; Monaghan [2000] and Belytschko *et al.* [2000]. Note that this work makes no attempt to resolve the above difficulties; rather, the scope of the discussion herein is limited to aspects pertaining only to 3D SPH applications in HVI problems.

The developed 3D SPH code follows the above-mentioned procedure. The stream of the code is schematically given in Fig. 1.

3. Taylor-Bar-Impact Test

To explore the effects corresponding to different model parameters during SPH simulation, the classic Taylor-Bar-Impact test [Taylor (1948)] is revisited. The test impacts a cylindrical bar, made of oxygen-free high-conductivity (OFHC) copper or Armco iron, perpendicularly onto a flat rigid surface.

3.1. Model setups

The cylinder, initially, is of length 25.46 mm and diameter 7.6 mm, and traveling at a speed of 221 m/s (Armco iron) or 190 m/s (OFHC copper). The test starts as the cylinders been contact with the rigid surface and concludes at $60 \,\mu s$, by then the kinetic energy would be dissipated over. For the 3D SPH discretization, the particles are initially "spheres" with a diameter of 0.64 mm, which results in a total of 4400 real particles in the model. Along the rigid surface, 1950 Monaghantype virtual particles are used to exert a repulsive boundary force to prevent the interior particles from penetrating. In addition, Libersky-type virtual particles are produced symmetrically outside the boundary in each evolution step for those real



Fig. 1. Flowchart of the SPH 3D code.

particles whose influencing domain reaches the boundary. The materials constitutive constants in the test follow those given in Johnson and Holmquist [1998], and are listed in Table 1 for the Johnson–Cook strength model and Table 2 for the Mie-Gruneisen equation of state, respectively.

Material	A(MPa)	B(MPa)	Ν	С	М	$T_{\rm melt}(^{\circ}{\rm C})$	$C_v(\mathrm{J/kg~C})$	Others
OFHC copper Armco iron	90 175	292 380	$\begin{array}{c} 0.31 \\ 0.32 \end{array}$	$0.025 \\ 0.060$	$1.09 \\ 0.55$	1083 1538	$383 \\ 452$	$\dot{\epsilon}_0 = 1.0 \mathrm{s}^{-1}$ $\dot{\epsilon}_{\min} = 0.002 \mathrm{s}^{-1}$

Table 1. Johnson–Cook strength model constants in Problem 1.

Table 2. Mie-Gruneisen EOS constants in Problem 1.

Material	$ ho_0({ m kg/m}^3)$	$C_s(\rm km/s)$	S_s	G(GPa)	ω	$J_0(MPa)$	Г
OFHC copper	8960	3.94	1.489	46	0.3	450	2.00
Armco iron	7890	3.63	1.800	80	0.3	600	1.81

3.2. Case definitions

A SPH simulation can be performed in many different settings. In core solutions, one may use different kernels, particle approximation formulations and time integration schemes. In subsidiary operations, implementation can vary in some aspects, such as nearest neighboring particle searching. In addition, one can choose various material strength models and equations of state according to the data available in the problems to solve. Some other numerical considerations, for instance, external force and physical viscosity effects, can also be taken into account during the simulation.

In this test, some scheme variations are examined to reveal possible relevant effects. Specifically, the material strength models tested include the Johnson–Cook model and elastic-perfectly-plastic model. The density can be evolved using either summation or continuity approach. Also, with the initial one-particle-per-hour, smoothing length can be adapted, using variable algorithm such as the one proposed by Monaghan and Lattanzio [1985], or simply fixed to be constant throughout the simulation. In addition, the sensitivity of the artificial viscosity parameters in the Monaghan-type viscosity is investigated. Cases of simulation with different combinations of above-mentioned variations are detailed in Table 3.

3.3. Result discussions

The purpose of this test is two-fold. The first is to examine the results of the standard cases (no. 7 and 14 for Armco iron and OFHC copper, respectively), with comparisons made to both experimental and numerical results from relevant refereed publications. The second objective of this revisit is to investigate various effects of different implementation schemes on the results.

3D rendering of the SPH calculated cylinder for the standard case of material Armco iron at the conclusion of the test is given in Fig. 2. A clear cylindrical symmetry is observed. For the standard case of material OFHC copper, evolution of the various energy components throughout the simulation is given in Fig. 3. It is seen that the kinetic energy is dissipated over at the conclusion of the test. Also, the conservation of the total energy is fairly good with little deviation. The plastic work

Case no.	Material	Density evolution	Strength model	Smoothing length	Viscosity parameters	Time stepping
1	Armco iron	Continuity	Johnson–Cook	Varied ^c	$\alpha = 1.0, \beta = 1.0$	Constant
2	Armco iron	Summation	Johnson–Cook	Varied	$\alpha = 1.0, \beta = 1.0$	Constant
3	Armco iron	Summation	Elastic-plastic ^b	Varied	$\alpha = 1.0, \beta = 1.0$	Constant
4	Armco iron	Continuity	Elastic-plastic	Varied	$\alpha = 1.0, \beta = 1.0$	Constant
5	Armco iron	Continuity	Elastic-plastic	Constant	$\alpha=1.0,\beta=1.0$	Constant
6	Armco iron	Continuity	Elastic-plastic	Varied	$\alpha = 1.0, \beta = 1.0$	Varied ^d
7^{a}	Armco iron	Continuity	Johnson–Cook	Constant	$\alpha=1.0,\beta=1.0$	Constant
8	OFHC copper	Continuity	Johnson–Cook	Varied	$\alpha=0.5,\beta=0.5$	Varied
9	OFHC copper	Continuity	Johnson–Cook	Varied	$\alpha=1.5,\beta=1.5$	Varied
10	OFHC copper	Continuity	Johnson–Cook	Varied	$\alpha = 2.0, \beta = 2.0$	Varied
11	OFHC copper	Continuity	Johnson–Cook	Varied	$\alpha=2.5,\beta=2.5$	Varied
12	OFHC copper	Continuity	Johnson–Cook	Varied	$\alpha=3.0,\beta=3.0$	Varied
13	OFHC copper	Continuity	Johnson–Cook	Varied	$\alpha=1.0,\beta=1.0$	Varied
$14^{\rm a}$	OFHC copper	Continuity	Johnson–Cook	Varied	$\alpha=1.0,\beta=1.0$	Constant

Table 3. Case definition in Problem 1.

 $^{\mathrm{a}}$ Standard case no. 7 and 14 for material Armco iron and OFHC copper, respectively;

^bLinearly elastic – perfectly plastic strength model;

 $^{\rm c}{\rm Monaghan}{\rm -Lattanzio}$ scheme which uses a smoothing length factor;

^dLibersky and Petschek scheme which is based on Courant–Friedrichs–Levy stable condition.



Fig. 2. 3D rendering of the deformed cylinder in Problem 1.

is found to be dominant during the impact, which consumes over 70% of the total initial kinetic energy in the end. In order to examine the results more precisely, the comparisons among the current solutions with both experimental and other numerical results are made, in terms of the length and the diameter of the deformed cylinder. As shown in Table 4, the current solution is in good agreement with the refereed results. It is worth noting that this revisit is not intended to simulate the



Fig. 3. Energy evolution in Problem 1.

Case no	Experimental result		EPIC solution		AUTODYN result		Current solution	
with material	L_{1}/L_{0}	R_1/R_0	L_{1}/L_{0}	R_1/R_0	L_{1}/L_{0}	R_1/R_0	L_{1}/L_{0}	R_{1}/R_{0}
7. Armco iron 14. OFHC copper	$0.780 \\ 0.638$	1.80 1.78	$0.755 \\ 0.682$	$1.80 \\ 1.92$	$0.744 \\ 0.697$	$1.38 \\ 1.57$	$0.716 \\ 0.696$	$1.74 \\ 1.80$

Table 4. Comparison of the deformed cylinders in Problem 1.

Note: Experimental result [Johnson and Holmquist (1998)]; EPIC solution [Zerilli and Armstrong (1987)]; AUTODYN result [Chin (2001)].

impact in a better accuracy (possible fine-tuning parameters), but rather to validate the SPH implementation. More importantly, this example is intended to reveal some effects from different SPH implementation schemes on the results yielded. The findings obtained from all cases are discussed below, in terms of the scheme variations proposed:

(1) Effect of density evolution approaches. The deformed cylinders for cases 1 and 2 at the conclusion of the testing are plotted in Fig. 4, corresponding to the so-called continuity and summation approach, respectively. For the summation approach (directly derived from the SPH particle approximation), the normalization process is found to be essential. Even though spurious results are observed on the impact surface, the diameter is unrealistically large. For problems with strong discontinuity, HVI in this case, the alternative continuity approach is preferred since it yields a better ability to handle material



(a) Continuity approach

(b) Summation approach

Fig. 4. Density evolution effect in Problem 1.



(a) Continuity approach: Johnson–Cook



(b) Continuity approach: Elastic-Perfect-Plastic



(c) Summation approach: Johnson–Cook (d) Summation approach: Elastic-Perfect-Plastic

Fig. 5. Strength model effect in Problem 1.

boundaries. Also, the continuity approach has a better computational efficiency in terms of the CPU time logged from the testing.

(2) Effect of material strength model. For both continuity and summation density evolution schemes, we implement both Johnson–Cook and elastic-perfect-plastic strength model (cases 1, 4 and 2, 3), with deformed cylinders plotted in Fig. 5. The simplified elastic-perfect-plastic model appears to be relatively stiffer as it yields cylinders of smaller diameters and larger heights. In addition, the featured strain-hardening effect in the Johnson–Cook model can be observed from the relatively clearer curve of budge effect of the cylinders. While more physically based strength models are pursued, the empirical Johnson–Cook model yields satisfactory results during this simulation. Comparison to the experiments' results suggests that the Johnson–Cook model may be too soft at large strains (results in a bigger diameter) and too hard at small strains (results in a thinner budge), which was also observed in [Zerilli and Armstrong (1987)].

- (3) Effect of smoothing length adaption. For both material strength models, we investigate the effect of the smoothing length adaption schemes (cases 1, 7 and 4, 5). It is found that keeping the smoothing length constant has made a small difference to the results from an adaptive scheme. When using the variable adaption scheme, we investigate the effect of the values of the smoothing length factor in the formula. It is found that increasing the factor, which turns out to be a larger smoothing length, has the effect of making the surface stiffer and produces a cylinder of a smaller diameter but of a bigger length. Note that the Monaghan scheme evolves the smoothing length according to a local number density of particles which relates to the initial particlization. Also, the change of the smoothing length significantly influences the computational efficiency since particle's interactions are altered. From the recorded information during simulation, doubling the smoothing length yields nearly 10 times the number of average interaction pairs.
- (4) Effect of artificial viscosity parameters. The Monaghan-type artificial viscosity was originally introduced to aim to diffuse sharp variations in the flow and dissipate the kinetic energy into heat in the shock front. Later, it was found helpful [Fulk (1997)] to stabilize SPH if properly specified. We investigate the numerical result's sensitivity to the parameters in the artificial viscosity (cases 8-13). It is found that a higher viscosity, especially artificial bulk viscosity associated with term α , decreases the diameter and increases the length of the final shape of the cylinder. In addition, the different values of parameters will alter the energy evolution situation quite much. More plastic work will be done in the case of a lower viscosity. Contrarily, with a higher viscosity, the proportion of artificial viscosity energy consumption increases and the total energy conservation becomes worse. The testing suggests a value close to one for both parameters.

4. Hypervelocity Impacts

This problem is modeled after a series of specimens tested by Hiermaier *et al.* [1997] at the Universität der Bundeswehr München. The test involved a normal HVI of a sphere on a thin plate, producing large deformation of structures. Unlike the original numerical simulation which employed the 2D planar symmetry of the problem, this test involves a 3D calculation since Hiermaier *et al.* concluded in their work that

A(MPa)	B(MPa)	Ν	С	М	$T_{\rm melt}(^{\circ}{\rm C})$	$C_v(\mathrm{J/kg}~\mathrm{C})$)	(Others	
300	426	0.34	0.015	1.0	502	875	$\dot{\epsilon}_0 =$	$= 1.0 \mathrm{s}^{-1}$	$, \dot{\epsilon}_{\min} =$	$= 0.002 \mathrm{s}^{-1}$
		18	able 6.	L'illotse	on EOS con	stants in Pr	oblem 2			
G	<i>P</i> 0	а	b	A_{T}	B_T	α_T	β_T	Energy	terms	(MJ/kg)
(GPa)	(kg/m^3)			(GPa	a) (GPa)	(GPa)	(GPa)	e_0	e_s	$e_{\rm sd}$
27.1	2790	0.5	1.63	75	65	5	5	5	3	15

Table 5. Johnson–Cook strength model constants in Problem 2.

a 3D simulation would probably deliver better results. Noticing that an oblique impact may produce interesting phenomena, we also performed the extended tests under different striking angles.

4.1. Model setups

Limited by the material data information, we only test the case of the impact of aluminum sphere on an aluminum plate. The sphere in high velocity is originally of 10 mm in diameter. The square plate is 4 mm thick and 40 mm wide. The impact speed of the sphere is 6.18 km/s. The room temperature and initial temperature of both sphere and plate are set to 0°C. A total of 25,242 particles are used in this simulation, with 2178 particles for the sphere and the rest of them for the plate. The test is run till $20 \,\mu s$ by which time the experimental results are available for comparison. Both the plate and sphere are free to move and deform, so there is no need to introduce the ghost particles for boundary conditions. In essence, the boundary effect is believed not to have propagated to the impact area during the instant $20 \,\mu s$, and therefore, the shock physics is not affected by this simplification while the computational price is significantly saved. Notably, the attempted use of the quarter symmetry involves the boundary conditions are not properly tackled.

Material properties and model constants follow those given in [Hiermaier *et al.* (1997)], detailed in Tables 5 and 6 for the employed Johnson–Cook strength model and the Tillotson equation of state [Tillotson (1962)], respectively.

4.2. Numerical considerations

The initial particle smoothing length is set in a way of *one-and-a-half* particles per hour. In the calculation of artificial viscosity, both parameters α and β are set to 2.5, which also follows Hiermaier's work for the purpose of comparison. To account for large deformation during the impact, a spatially and temporally variable smoothing length will be necessary. In this case, Benz's [1990] scheme, which takes the time derivative of the smoothing function in terms of the continuity equation, is selected to evolve the smoothing length.

In addition, the cubic-spline kernel function [Monaghan and Lattanzio (1985)], tree algorithm [Hernquist and Katz (1989)] for nearest neighbor particle searching, continuity density approach [Monaghan (1992)] and SPH particle approximation specified in Eq. (1) are used in this simulation.

4.3. Result discussions

Figure 6 shows the total energy conservation throughout the analysis. This fairly good energy conservation, with a low deviation, underlines the reliability of the results. Numerically, the results obtained from the simulation compares, given in Table 7, favorably with the experiment in that the size of the crater and the shape of the debris cloud both agree well. As seen, 3D calculations from this testing, compared to 2D cases, greatly improve the results, on both the measurements of the crater and the debris cloud.

The plots of the impact at a list of durations of 1, 5, 10, 15, and $20 \,\mu s$ are given in Figs. 7, 8, and 9, corresponding to top, side, and 3D rendering viewpoints, respectively. It is clearly seen that the impact at high velocity results in big craters, with the debris clouds behind the target plate. The plots from different viewpoints clearly depict the contours of the progressive interface for both the target and the



Fig. 6. Energy evolution in Problem 2.

Simulation	Crater diameter (mm)	Debris cloud length l (mm)	Debris cloud width w (mm)	Ratio (shape) of debris cloud: l/w
Present	$31.6\sim 35.3^{\rm c}$	102.8	75.5	1.36
Experimental ^a	$27.5\sim34.5^{\rm c}$		_	1.39
Numerical ^a	35.0		_	1.11
Numerical ^b	28.9	105.1	86.1	1.22

Table 7. Comparison of HVI simulation results in Problem 2.

^aRef. [Hiermaier *et al.* (1997)];

^bRef. [Chin (2001)];

^cIncluding/Excluding crater lip.

projectile. The cylindrical symmetry is observed from all plots. The non-physical sparks (in the top view) are believed to be inherited from the initial particlization, which is generated by a layered algorithm. Overall, the results obtained show that SPH is well suited for the desired impact problems.

4.4. Extended tests

Noticing that an oblique impact may produce interesting phenomena, we also performed the extended tests on impacts under different striking angles. The degrees of angle tested include 15, 30, 45, 60, and 75, with a degree of 90 being the original normal impact. All testing parameters are exactly same as those employed in the perpendicular case except that the geometry of the plate is increased in order to capture a more complete deformation shape.

The plots of impact at conclusion of the test $(20 \,\mu s)$ for all different striking angles are given in Figs. 10, 11 and 12, corresponding to top, side and 3D rendering view perspectives, respectively. Through the comparisons among tests, some observations are discussed below.

- From the top view of the plots, it is observed that the shape of the crater is no longer a circle. When the striking angle becomes bigger, the shape of the crater becomes closer to a circle.
- From the side view of the plots, it is seen that the impact with a very small striking angle scratches, rather than impacts, the target plane. For example, in the case of 15°, almost all sphere particles are reflected backward instead of penetrating forward. As the striking angle becomes more normal to the plane, the phenomenon of penetration becomes clearer.
- From the 3D rendering view of the plots, it can be seen that the shape of the debris cloud changes with different striking angles. A more perpendicular impact produces a larger debris cloud, in both width and length. However, it is interesting to find that the ratio of the debris cloud (length over width) remains a value close to 1.3 within all tested cases.





Fig. 7. Top view of normal HVI in Problem 2.

• Unlike the normal impact, oblique impacts lose the cylindrical symmetry during the impacts. From the side view of the plots, one can readily see that the particles of sphere (in green) deposit more densely in the direction of the impact. However, one can easily find that they still possessed one symmetry plane, the



Fig. 8. Side view of normal HVI in Problem 2.



Fig. 9. 3D rendering view of normal HVI in Problem 2.



0.

0.06



Fig. 10. Top view of oblique HVI in Problem 2.



Fig. 11. Side view of oblique HVI in Problem 2.



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Fig. 12. 3D rendering view of oblique HVI in Problem 2.

plane through the center of the sphere normal to the target plane and in the direction of the impact.

While no experimental data for oblique impact are available, the particle evolution obtained likely demonstrates that the current procedure is capable of capturing the major features of such impacts.

5. Conclusions

In this paper, a three-dimensional computational simulation of the HVI using SPH is presented. For both normal and oblique HVI, the good representation of crater and debris cloud shows that SPH is a promising candidate for HVI problems. Test findings described in this paper represent a database useful for investigating the current 3D SPH procedure for HVI problems. As well, they serve to emphasize the behavioral concepts important to understanding and correctly modeling the HVI problems using the conventional SPH method. While the damage attempt is not included in this simulation, the need to incorporate a damage and fracture model into the current SPH procedure is recognized and currently on-going for this implementation.

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